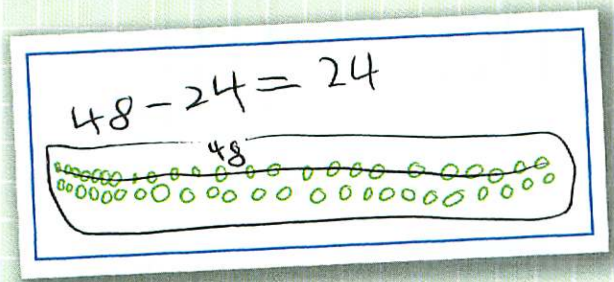
Teacher Note

Notating Subtraction Strategies

A challenge that students face as they develop and refine their strategies for subtraction in Grade 2 and beyond is finding clear and efficient ways to communicate their mathematical thinking on paper. Second graders have several models and representations available to them as they record their work, including drawings, the 100 chart, the number line, sticker notation, numbers, and equations.

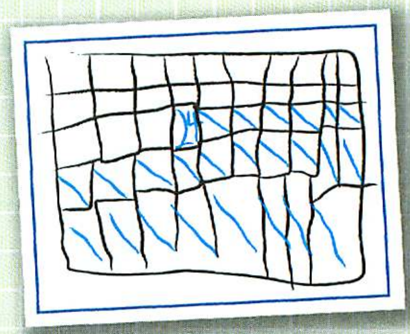
Drawing All and Removing Some

A few students may draw all in a way that does not make efficient use of groups, cross out the amount subtracted, and then count those that remain. This is a time-consuming strategy for solving 2-digit subtraction problems and recording work. For example, Anita solves 48 – 24 by drawing 48 circles, crossing out 24, and counting those that remain. This strategy entails a great deal of counting and recounting.



Anita's Work

Malcolm used a more efficient strategy to solve the problem. He used cubes in towers of 10, but when faced with the task of recording his work, he drew individual cubes—a challenging and time-consuming drawing task.

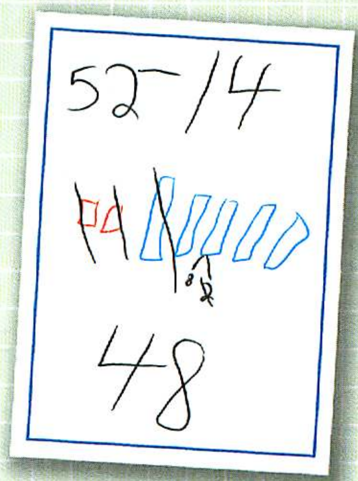


Malcolm's Work

Help students like Anita see how they can use cubes in towers of 10 or sets of paper stickers to represent the 48 and remove the 24 more efficiently. Encourage all students in this category to consider using sticker notation to record their work.

Sticker Notation

Many second graders use stickers or cubes in towers of 10 to solve subtraction problems. Using sticker notation to record what they did can be challenging, particularly when the number to be subtracted has more ones than the number they are subtracting from. Ending up with 10 too many, as Katrina and Esteban did, is a common error.

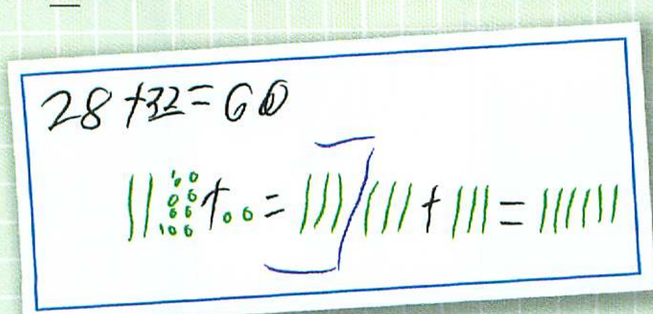


Katrina's Work

Esteban's Work

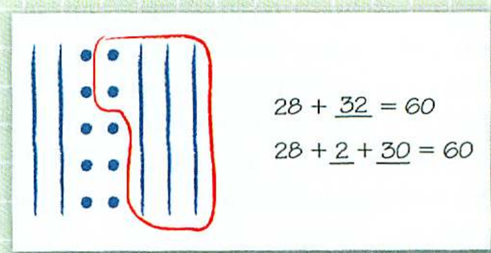
These students forget to cross out the strip of 10 when they break it into parts or trade it in for 10 singles. For example, Esteban draws 45 stickers and crosses out two strips and five singles. Because he still needs to subtract 2, he breaks one of the remaining strips into 2 and 8 and crosses out the 2. When he counts the stickers that remain, he counts that strip as both a 10 *and* an 8 because he did not cross it out when he broke it up. Therefore, he ends up with an answer of 28 (10 + 10 + 8) instead of 18. Ask these students to explain the steps they took to solve the problem so that you can discuss what happens with them at this point in the problem.

Also be on the lookout for students who painstakingly show every step and/or every amount in the form of stickers. For example, Juan adds up to solve 60 – 28. He shows that 28 + 2 = 30 and 30 + 30 = 60. Therefore, 28 + <u>32</u> = 60.

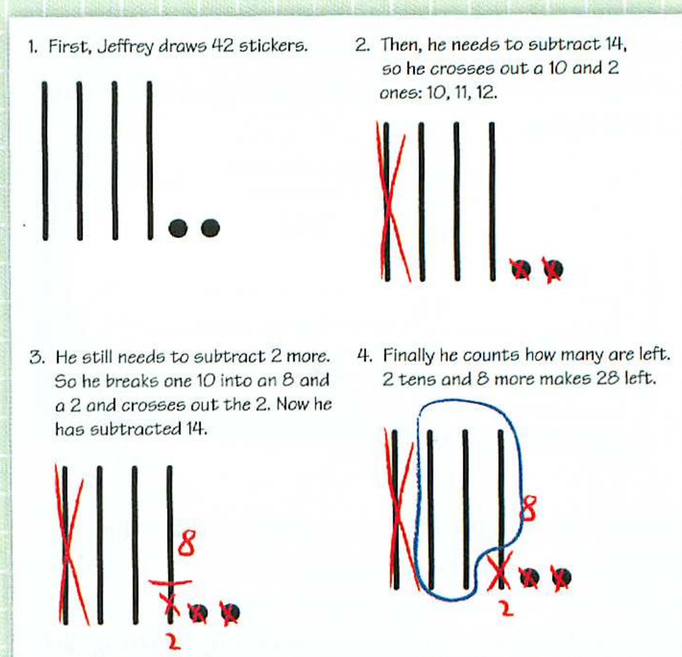


Juan's Work

As Juan becomes more fluent with this strategy and sees it modeled by his teacher and other students, he will develop more efficient ways of using stickers and then equations to represent his work.



Although not expected of students, be sure that when you use stickers to model strategies for the class, you do so step by step. For example, Jeffrey works through four steps as he uses stickers to represent his strategy for subtracting 14 from 42.

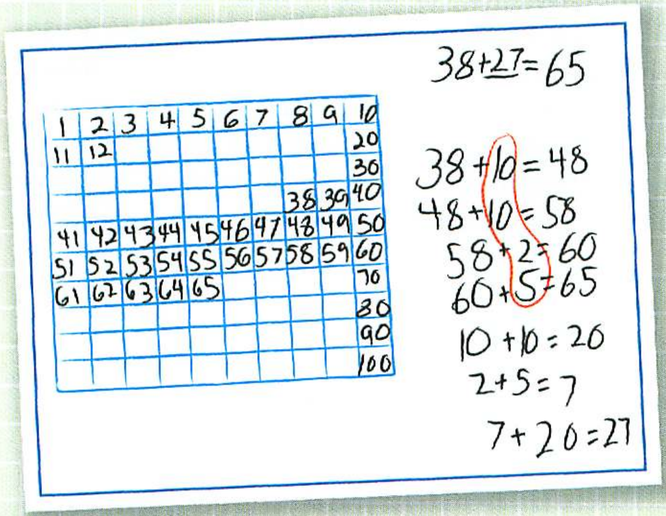


The image shown in Step 4 alone should be considered a complete piece of student work; it contains all the steps Jeffrey went through.

The 100 Chart

Many students who use the 100 chart (or a sheet of 100 stickers) to solve subtraction problems have difficulty recording what they did on paper. Some say, "I used a 100 chart" or "I counted on the 100 chart." Others try to draw the chart in whole or part. Encourage these students to describe the math of their strategy in writing; for example, "You said you counted on the 100 chart. *How* did you count? Did you count by ones? By tens?"

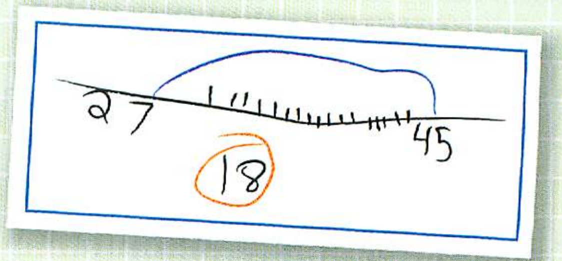
When students can do that, focus on helping them write equations and on connecting those equations to the jumps they are making on the 100 chart. For example, when the teacher visited Melissa, she was using a page in a sticker book to solve 38 + ____ = 65. After helping Melissa see that she did not need to fill in every number and working through the problem with her, the teacher helped her use equations to record her work by asking questions such as, "So you added up to find the difference between 38 and 65. You said you started at 38 and jumped 10. Where did you land? What did you do next?"



Melissa's Work

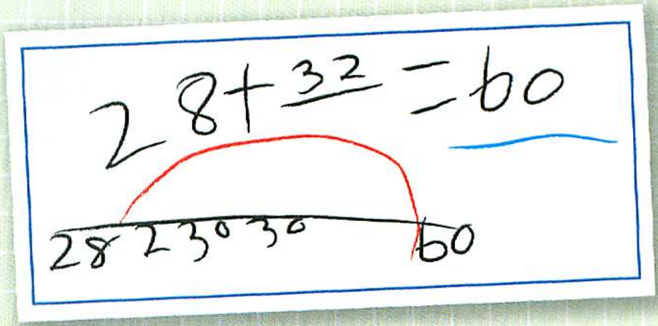
The Number Line

Many second graders use the number line to solve subtraction problems and to show how they solved them. For some students, drawing and labeling a number line and then showing a solution on it can be a challenging fine-motor task. For that reason, some teachers provide small prepared number lines for students to use to record their work.



Nate's Work

For other students, the challenge is showing how they subtracted or added up, how they know they subtracted or added all that they needed to, and how to make their answer clear—all in such a way that someone else can read and understand their strategy. For example, to solve 60 – 28, Darren added up to find the difference. He thought, "28 + 2 = 30 and then 30 + 30 = 60. So, I had to add 32 to 28 to get 60."



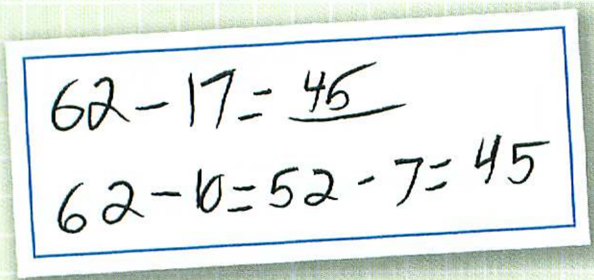
Darren's Work

Darren is still learning the conventions of recording a strategy on the number line. Seeing his strategy modeled by his teacher and by classmates who use the number line will help him develop clearer methods for recording his work.

Numbers, Equations, and Vertical Notation

At this point in the year, many students use numbers and equations to show their strategies. There are some typical issues to look for as students use numbers and equations to record their work.

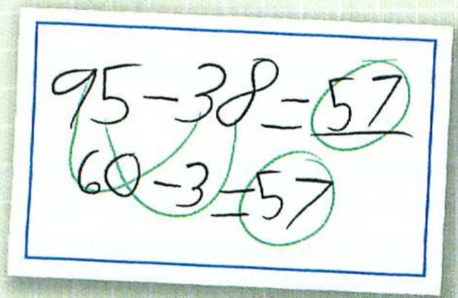
The run-on sentence Because students have been thinking about the meaning of the equal sign all year long, particularly during the course of *Today's Number,* you should not see many who record in this way.



Tia's Work

It is clear that Tia is saying she subtracted 10 from 62 and then 7 from 52, but 62 – 10 does *not* equal 52 – 7. Question students who record in such a way, focusing on equality and what the equal sign means. For example, "I see here that you have an equation that says 62 – 10 = 52 – 7. Does 62 – 10 *equal* 52 – 7?"

Lines, carets, and arrows Some students who use lines, carets, or arrows to show their strategy for adding 2-digit numbers try to transfer that strategy to subtracting 2-digit numbers. As with addition, clarity and efficiency can be a problem with these recording methods. Subtraction, however, presents an additional difficulty.

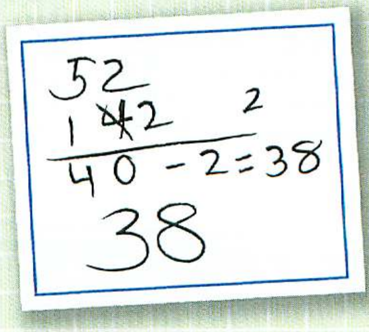


Travis's Work

Although Travis's work looks neat, gives a pretty clear sense of his strategy, and gives the right answer, consider the explanation he gave his teacher (90 − 30 = 60, 8 − 5 = 3, and 60 − 3 = 57). For this reason, encourage these students to use horizontal equations, and do not use lines, carets, and arrows to model strategies for the class.

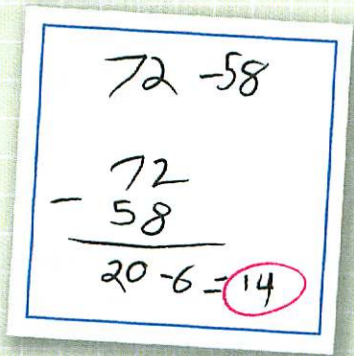
Vertical notation It is important that students be comfortable solving subtraction problems presented either horizontally or vertically by the end of Grade 2. Vertical notation does not suggest a particular strategy. Students should not see the way a problem is presented as a directive to carry out a particular strategy. Be on the lookout for students who believe that when they see a problem written vertically, they must carry out a particular algorithm rather than consider what they know about the numbers in the problem.

Most students record their strategies by using one of the above methods and/or horizontal equations. If you have students who are experimenting with ways to use vertical notation to record their strategies, there are some common issues to watch for. For example, Yama subtracts 10 from 50 and gets 40. She writes 4 in the tens place and then considers the ones. Because she cannot subtract 4 from 2, she subtracts 2, writes 0 in the ones place, and reminds herself that she still needs to subtract 2 more by writing another 2 to the right of the original problem. Then she subtracts that 2 from the 40 and gets her answer of 38.



Yama's Work

Students like Yama compute accurately and use a strategy that is mathematically sound and makes sense to them. However, they need support finding a way to record their work clearly and efficiently.



Simon's Work

Simon uses vertical notation to record a strategy much like Travis's.

$$72 - 58 =$$

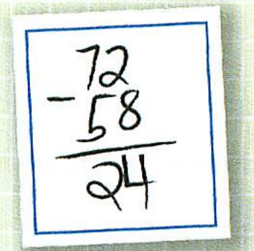
$$70 - 50 = 20$$

$$2 - 8 = -6$$

$$20 + -6 = 14$$

Ask students who use this notation to explain how they solved the problem. Many students give problematic explanations like Travis's. Others may say, "70 − 50 = 20. You can't take 8 from 5, so that's 0. So my answer is 20." These strategies highlight the issues that come up when subtracting by place and the difficulty second graders have making sense of it.

Other students who use vertical notation are attempting to use the U.S. standard algorithm for subtraction.



Alberto's Work

When you find students who are misapplying this strategy or using it without meaning, help them make sense of and practice a strategy that relies on what they know about the operation of subtraction and the quantities in the problem and offers a clear and efficient way to notate their work, such as subtracting in parts or adding up.